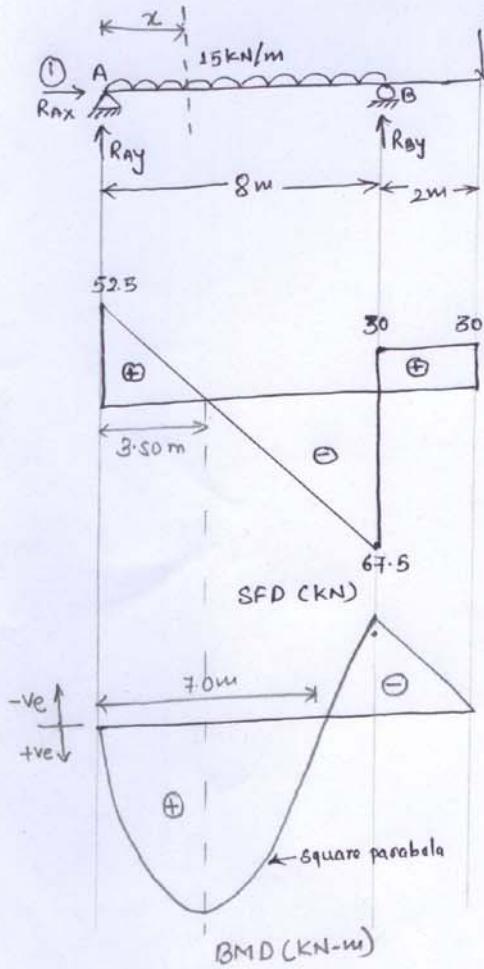


## Beam Tutorial



### Calculation of support Reaction:

$$\begin{aligned} (\rightarrow) \sum F_x &= 0 \\ R_{Ax} &= 0 \end{aligned}$$

$$\begin{aligned} (\uparrow) \sum F_y &= 0 \\ R_{Ay} + R_{By} - 15 \times 8 - 30 &= 0 \\ \text{or } R_{Ay} + R_{By} &= 150 \quad \text{--- (1)} \end{aligned}$$

$$\begin{aligned} (\curvearrowright) \sum M_A &= 0 \\ 8R_{By} - 15 \times 8 \times \frac{8}{2} - 30 \times 10 &= 0 \\ R_{By} &= 97.5 \text{ kN } (\uparrow) \\ \therefore R_{Ay} &= 150 - 97.5 = 52.5 \text{ kN } (\uparrow) \end{aligned}$$

### Shear Force Calculation $\left( \begin{array}{c} \uparrow +ve \\ \downarrow -ve \end{array} \right)$

$$\begin{aligned} V_{AL} &= 0 \\ V_{AR} &= 52.5 \text{ kN} \\ V_{BL} &= V_{AR} - 15 \times 8 = -67.5 \text{ kN} \\ V_{BR} &= V_{BL} + 97.5 = 30 \text{ kN} \\ V_{CL} &= V_{BR} = 30 \text{ kN} \\ V_{CR} &= V_{CL} - 30 = 0 \end{aligned}$$

### Bending Moment Calculation $\left( \begin{array}{c} \curvearrowright +ve \\ \curvearrowleft -ve \end{array} \right)$ :

$$\begin{aligned} M_A &= 0 \\ M_B &= 52.5 \times 8 - 15 \times 8 \times \frac{8}{2} = -60 \text{ kN-m} \end{aligned}$$

$$M_C = 52.5 \times 10 - 15 \times 8 \times \left(2 + \frac{8}{2}\right) = 0$$

For Point of Contraflexure (In portion AB)

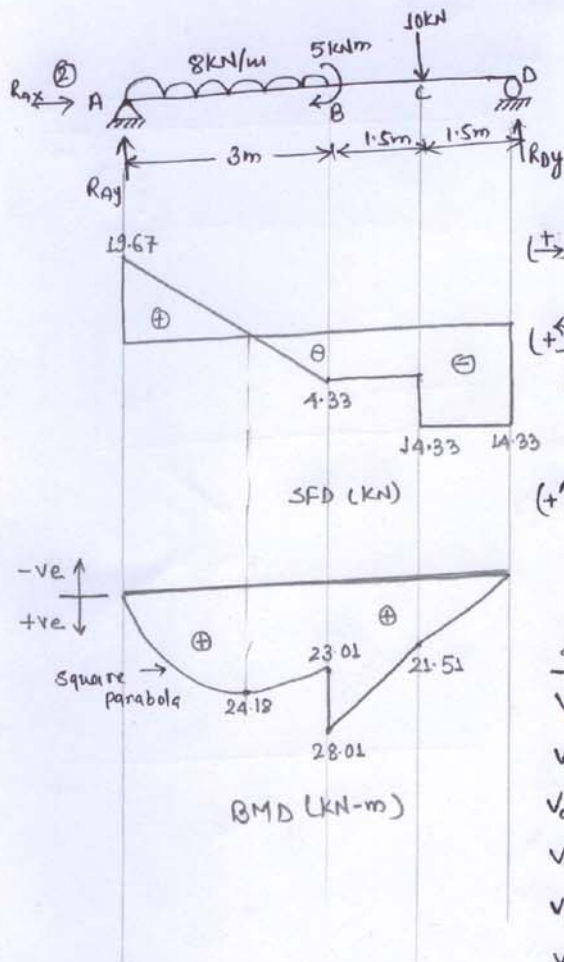
$$\begin{aligned} M_x &= 52.5x - 15 \times x \times \frac{x}{2} = 0 \\ \Rightarrow x &= 7.0 \text{ m} \end{aligned}$$

For SF zero point (In portion AB)

$$\begin{aligned} \frac{52.5}{x} &= \frac{67.5}{8-x} \\ \therefore x &= 3.5 \text{ m} \end{aligned}$$

$\therefore$  maximum moment at  $x = 3.5 \text{ m}$

$$\begin{aligned} \text{is } M_{\max} &= 52.5 \times 3.5 - 15 \times 3.5 \times \frac{3.5}{2} \\ &= 91.875 \text{ kN-m} \end{aligned}$$



Calculation of support  $R_x$  :-

$$(\rightarrow) \sum F_x = 0$$

$$R_{Ax} = 0$$

$$(\curvearrowright) \sum M_A = 0$$

$$6R_{Dy} - 5 - 10 \times 4.5 - 8 \times 3 \times \frac{3}{2} = 0$$

$$R_{Dy} = 14.33 \text{ kN}$$

$$(\uparrow) \sum F_y = 0$$

$$R_{Ay} + R_{Dy} - 10 - 8 \times 3 = 0$$

$$R_{Ay} = 19.67 \text{ kN}$$

Shear Force Calculation ( $\uparrow$  +ve  $\downarrow$ )

$$V_A = R_{Ay} = 19.67 \text{ kN}$$

$$V_B = 19.67 - 8 \times 3 = -4.33 \text{ kN}$$

$$V_{CL} = V_B = -4.33 \text{ kN}$$

$$V_{CR} = V_{CL} - 10 = -14.33 \text{ kN}$$

$$V_{DL} = V_{CR} = -14.33 \text{ kN}$$

$$V_{DR} = V_{DL} + R_{Dy} = 0$$

Bending Moment Calculation ( $\curvearrowright$  +ve  $\curvearrowleft$ )

$$M_A = 0$$

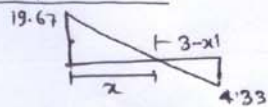
$$M_{BL} = 19.67 \times 3 - 8 \times 3 \times \frac{3}{2} = 23.01 \text{ kN-m}$$

$$M_{BR} = 23.01 + 5 = 28.01 \text{ kN-m}$$

$$M_C = 19.67 \times 4.5 - 8 \times 3 \times 3 + 5 = 21.51 \text{ kN-m}$$

$$M_D = 19.67 \times 6 - 8 \times 3 \times (3 + \frac{3}{2}) + 5 - 10 \times 1.5 = 0$$

For shear force zero point  
In portion AB:



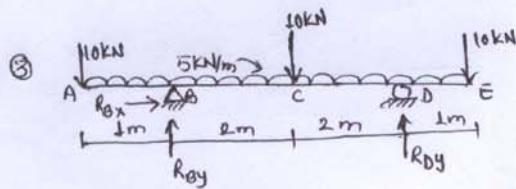
$$\frac{x}{19.67} = \frac{3-x}{4.33}$$

$$\Rightarrow x = 2.458 \text{ m}$$

$\therefore$  maximum bending moment  
in section AB occurs at 2.458 m  
from A

$$M_{\max} = 19.67 \times 2.458 - 8 \times 2.458 \times \frac{2.458}{2} = 24.18 \text{ kN-m}$$

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Due to symmetry

$$R_{Ay} = R_{Dy} = \frac{(10+10+10+5 \times 6)}{2} = 30 \text{ kN}$$

Shear force Calculation ( $\uparrow$  +ve,  $\downarrow$  -ve)

$$V_{AL} = 0$$

$$V_{AR} = -10 \text{ kN}$$

$$V_{BL} = V_{AR} - 5 \times 1 = -15 \text{ kN}$$

$$V_{BR} = V_{BL} + 30 = 15 \text{ kN}$$

$$V_{CL} = V_{BR} - 5 \times 2 = 5 \text{ kN}$$

$$V_{CR} = V_{CL} - 10 = -5 \text{ kN}$$

$$V_{DL} = V_{CR} - 5 \times 2 = -15 \text{ kN}$$

$$V_{DR} = V_{DL} + R_{Dy} = 15 \text{ kN}$$

$$V_{EL} = V_{DR} - 5 \times 1 = 10 \text{ kN}$$

$$V_{ER} = V_{EL} - 10 = 0$$

Bending Moment Calculation ( $\curvearrowright$  +ve)

$$M_A = 0$$

$$M_B = -10 \times 1 - 5 \times 1 \times \frac{1}{2} = -12.5 \text{ kN-m}$$

$$M_C = -10 \times 3 - 5 \times 3 \times \frac{3}{2} + 30 \times 2 = 7.5 \text{ kN-m}$$

$$M_D = M_B = -12.5 \text{ kN-m}$$

$$M_E = 0$$

$$M_{AB, \text{mid}} = -10 \times \frac{1}{2} - 5 \times \frac{1}{2} \times \frac{1}{2} \times 0.5 = -5.625 \text{ kN-m}$$

$$M_{DE, \text{mid}} = 5.625 \text{ kN-m}$$

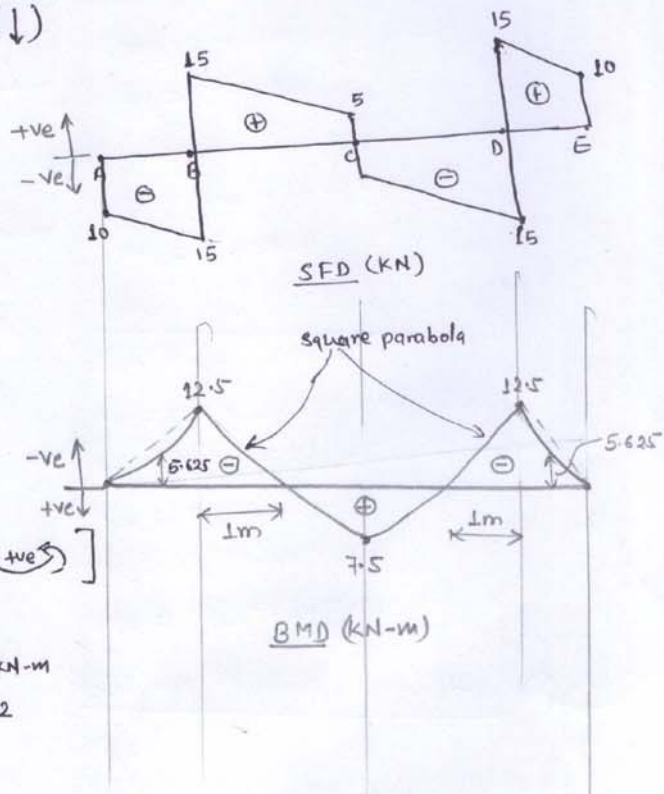
$$M_{BC, \text{mid}} = -10 \times 2 - 5 \times 2 \times \frac{2}{2} + 30 \times 1 = 0$$

For point of contraflexure in portion BC (origin at B)

$$M_x = -10(x+1) - 5(1+x) \times \frac{(1+x)}{2} + 30x = 0$$

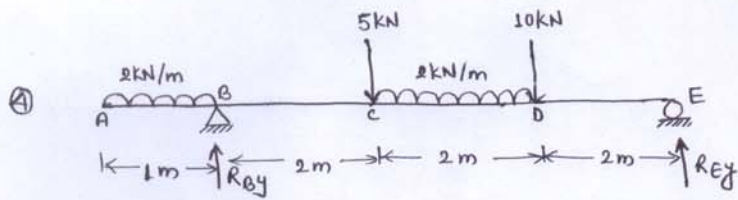
$$\therefore x = 1 \text{ m}$$

Similarly for portion  $x = 1 \text{ m}$  from point D.



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Calculation of Support  $R_x$ :

$$(+\curvearrowright) \sum M_B = 0$$

$$6R_{Ey} + 2 \times 1 \times \frac{1}{2} - 5 \times 2 - 10 \times 4 - 2 \times 2 \times (2 + \frac{2}{2}) = 0$$

$$R_{Ey} = 10.16 \text{ kN}$$

$$(+\uparrow) \sum F_y = 0$$

$$R_{By} + R_{Ey} - 5 - 10 - 2 \times 2 - 2 \times 1 = 0$$

$$R_{By} = 10.83 \text{ kN}$$

Shear Force calculation ( $\uparrow$  +ve,  $\downarrow$  -ve)

$$V_A = 0$$

$$V_{BL} = V_A - 2 \times 1 = -2 \text{ kN}$$

$$V_{BR} = V_{BL} + 10.83 = 8.83 \text{ kN}$$

$$V_{CL} = V_{BR} = 8.83 \text{ kN}$$

$$V_{CR} = V_{CL} - 5 = 3.83 \text{ kN}$$

$$V_{DL} = V_{CR} - 2 \times 2 = -0.17 \text{ kN}$$

$$V_{DR} = V_{DL} - 10 = -10.17 \text{ kN}$$

$$V_{EL} = V_{DR} = -10.17 \text{ kN}$$

$$V_{ER} = V_{EL} + R_{Ey} = 0$$

Bending Moment Calculation ( $\curvearrowright$  +ve)

$$M_A = 0$$

$$M_B = -2 \times 1 \times \frac{1}{2} = -1 \text{ kN-m}$$

$$M_C = -2 \times 1 \times (2 + \frac{1}{2}) + 10.83 \times 2 = 16.66 \text{ kN-m}$$

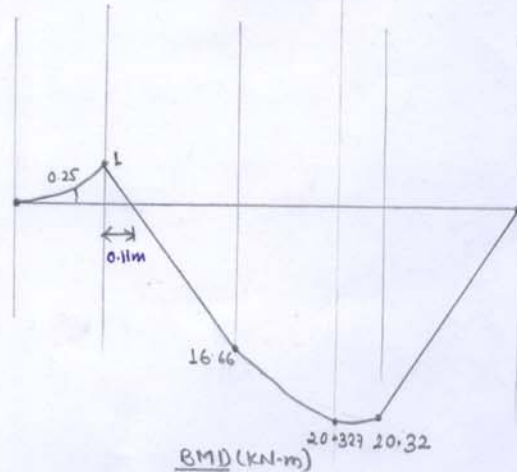
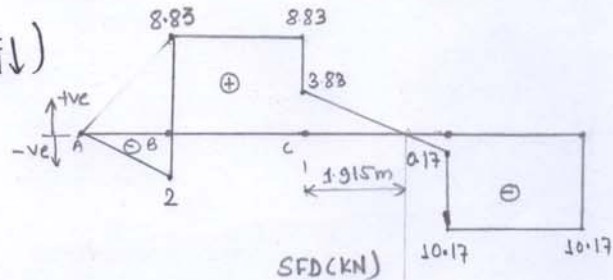
$$M_D = -2 \times 1 \times (4 + \frac{1}{2}) + 10.83 \times 4 - 5 \times 2 - 2 \times 2 \times \frac{2}{2} = 20.32 \text{ kN-m}$$

$$M_E = 0$$

$$M_{AB, mid} = -2 \times 0.5 \times \frac{0.5}{2} = -0.25 \text{ kN-m}$$

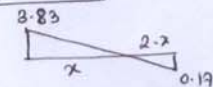
For Point of contraflexure in BC

$$\frac{1}{x} = \frac{16.66}{2-x} \Rightarrow x = 0.11 \text{ m}$$

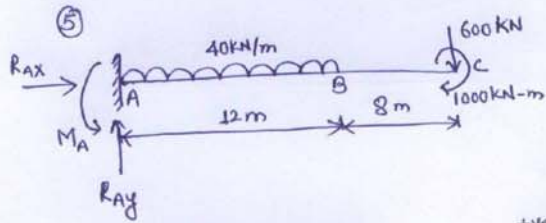


For SF zero point in portion CD

$$\frac{x}{3.83} = \frac{2-x}{0.17} \Rightarrow x = 1.915 \text{ m}$$



$$\therefore \text{max}^m \text{ BM } (M_{max}) = -2 \times 1 \times (3.915 + \frac{1}{2}) + 10.83 \times 3.915 - 5 \times 1.915 - 2 \times 1.915 \times \frac{1.915}{2} = 20.327 \text{ kN-m}$$



Calculation of reaction:

$$(\rightarrow) \sum F_x = 0$$

$$R_{Ax} = 0$$

$$(\uparrow) \sum F_y = 0$$

$$R_{Ay} - 40 \times 12 - 600 = 0$$

$$R_{Ay} = 1080 \text{ kN}$$

$$(\curvearrowleft) \sum M_A = 0$$

$$M_A - 40 \times 12 \times \frac{12}{2} - 1000 - 600 \times 20 = 0$$

$$M_A = 15880 \text{ kN-m}$$

Shear force calculation ( $\uparrow$  +ve,  $\downarrow$  -ve)

$$V_{AL} = 0$$

$$V_{AR} = 1080 \text{ kN}$$

$$V_B = V_{AR} - 40 \times 12 = 600 \text{ kN}$$

$$V_{CL} = V_B = 600 \text{ kN}$$

$$V_{CR} = V_{CL} - 600 = 0$$

Bending Moment calculation ( $\curvearrowright$  +ve)

$$M_{AL} = 0$$

$$M_{AR} = -15880 \text{ kN-m}$$

$$M_B = -15880 + 1080 \times 12 - 40 \times 12 \times 6$$

$$= -5880 \text{ kN-m}$$

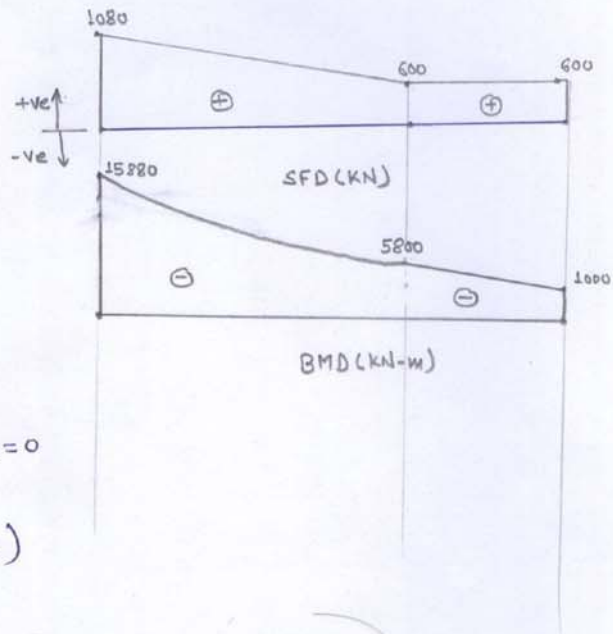
$$M_{AB, \text{mid}} = -15880 + 1080 \times 6 - 40 \times 6 \times \frac{6}{2}$$

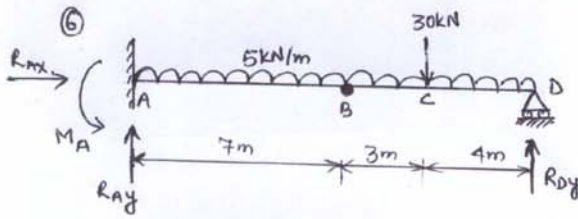
$$= -10120 \text{ kN-m}$$

$$M_{CL} = -15880 + 1080 \times 20 - 40 \times 12 \times \left(8 + \frac{12}{2}\right)$$

$$= -1000 \text{ kN-m}$$

$$M_{CR} = M_{CL} + 1000 = 0$$





Calculation of Support Reaction:

$$\sum M_{B, \text{right}} = 0$$

$$7R_{Dy} - 30 \times 3 - 5 \times 7 \times \frac{7}{2} = 0$$

$$R_{Dy} = 30.36 \text{ kN}$$

$$\sum F_y = 0$$

$$R_{Ay} + R_{Dy} - 30 - 5 \times 14 = 0$$

$$R_{Ay} = 69.64 \text{ kN}$$

$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$\sum M_A = 0$$

$$M_A - 5 \times 14 \times 7 - 30 \times 10 + 30.36 \times 14 = 0$$

$$M_A = 364.96 \text{ kN-m}$$

Shear Force Calculation ( $\uparrow$  tve,  $\downarrow$ )

$$V_{AL} = 0$$

$$V_{AR} = 69.64 \text{ kN}$$

$$V_{CL} = V_{AR} - 5 \times 10 = 19.64 \text{ kN}$$

$$V_{CR} = V_{CL} - 30 = -10.36 \text{ kN}$$

$$V_{DL} = V_{CR} - 5 \times 4 = -30.36 \text{ kN}$$

$$V_{DR} = V_{DL} + R_{Dy} = 0$$

Bending moment calculation ( $\curvearrowright$  tve,  $\curvearrowleft$ )

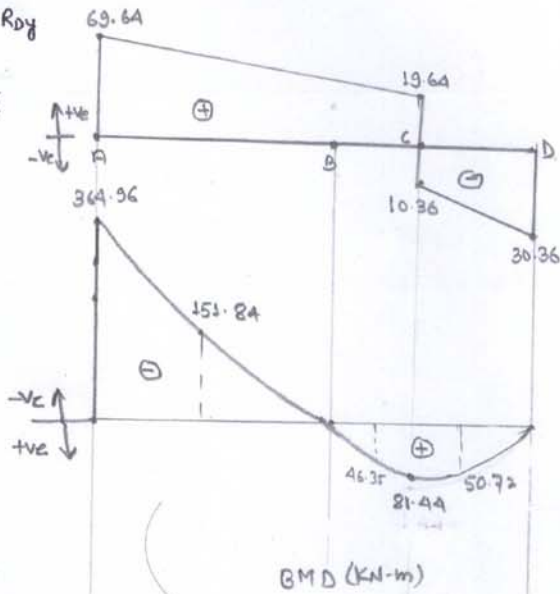
$$M_{AL} = 0$$

$$M_{AR} = -364.96 \text{ kN-m}$$

$$M_B = -364.96 + 69.64 \times 7 - 5 \times 7 \times \frac{7}{2} = 0$$

$$M_C = -364.96 + 69.64 \times 10 - 5 \times 10 \times \frac{10}{2} = 81.44 \text{ kN-m}$$

$$M_D = -364.96 + 69.64 \times 14 - 5 \times 14 \times \frac{14}{2} - 30 \times 4 = 0$$



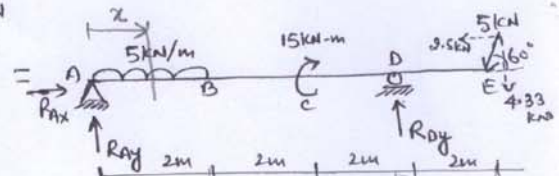
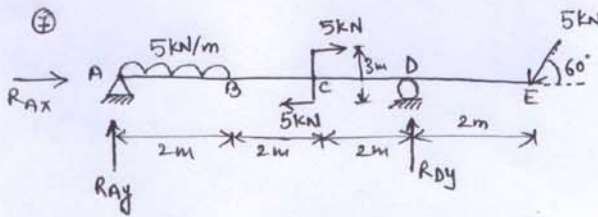
$$M_{AB, \text{mid}} = -364.96 + 69.64 \times 3.5 - 5 \times 3.5 \times \frac{3.5}{2} = -151.84 \text{ kN-m}$$

$$M_{CD, \text{mid}} = 30.36 \times 2 - 5 \times 2 \times \frac{2}{2} \text{ (from right side)} = 50.72 \text{ kN-m}$$

$$M_{BC, \text{mid}} = -364.96 + 69.64 \times (7 + \frac{3}{2}) - 5 \times (7 + 1.5) \times \frac{(7 + 1.5)}{2} = 46.35 \text{ kN-m}$$

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Calculation of Support reaction:-

$(\pm) \sum F_x = 0$

$R_{Ax} - 5 \cos 60 = 0$

$R_{Ax} = 2.5 \text{ kN}$

$(+\curvearrowright) \sum M_A = 0$

$-5 \times 2 \times \frac{2}{2} - 15 + 6 R_{Dy} - 8 \times 5 \sin 60 = 0$

$R_{Dy} = 9.94 \text{ kN}$

$(\uparrow) \sum F_y = 0$

$R_{Ay} + R_{Dy} - 5 \times 2 - 4.33 = 0$

$R_{Ay} = 4.40 \text{ kN}$

Calculation of Axial Force ( $\leftarrow$  +ve  $\rightarrow$ )

$AF_{AB} = -2.5 \text{ kN}$

Calculation of shear Force ( $\uparrow$  +ve  $\downarrow$ )

$V_{AL} = 0$

$V_{AR} = 4.40 \text{ kN}$

$V_B = V_{AR} - 5 \times 2 = -5.60 \text{ kN}$

$V_{DL} = V_B = -5.6 \text{ kN}$

$V_{DR} = V_{DL} + 9.94 = 4.34 \text{ kN}$

$V_{EL} = V_{DR} = 4.34 \text{ kN}$

$V_{ER} = V_{EL} - 4.33 = 0$

Calculation of Bending Moment ( $\curvearrowright$  +ve  $\curvearrowleft$ )

$M_A = 0$

$M_B = 4.4 \times 2 - 5 \times 2 \times 1 = -1.20 \text{ kN-m}$

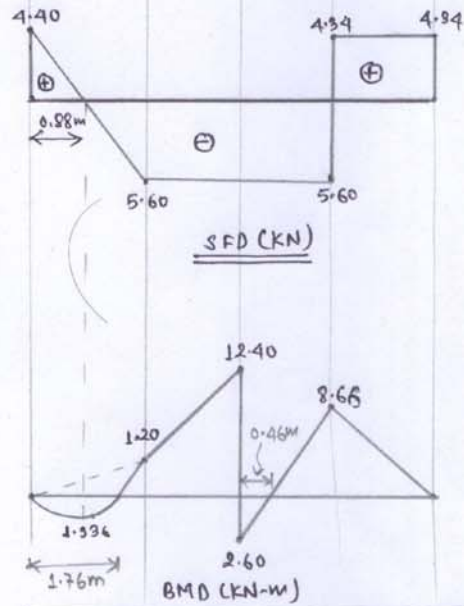
$M_{CL} = 4.4 \times 4 - 5 \times 2 \times (2+1) = -12.4 \text{ kN-m}$

$M_{CR} = M_{CL} + 15 = 2.60 \text{ kN-m}$

$M_D = -4.33 \times 2 = -8.66 \text{ kN-m}$  (from right side)

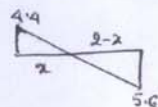
$M_E = 0$

$M_{AB, \text{mid}} = 4.4 \times 1 - 5 \times 1 \times 0.5 = 1.90 \text{ kN-m}$



For SF zero point in AB

$\frac{x}{4.4} = \frac{2-x}{5.6}$   
 $\Rightarrow x = 0.88 \text{ m}$



$\therefore$  max<sup>m</sup> BM at  $x = 0.88$  is

$M_{\text{max}} = 4.40 \times 0.88 - 5 \times 0.88 \times \frac{0.88}{2}$   
 $= 1.936 \text{ kN-m}$

For point of contraflexure

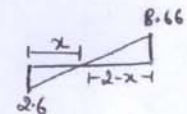
a) In portion AB

$M_x = 4.40x - 5x \times \frac{x}{2} = 0$

$\Rightarrow x = 0, 1.76$   
 i.e.  $x = 1.76 \text{ m}$

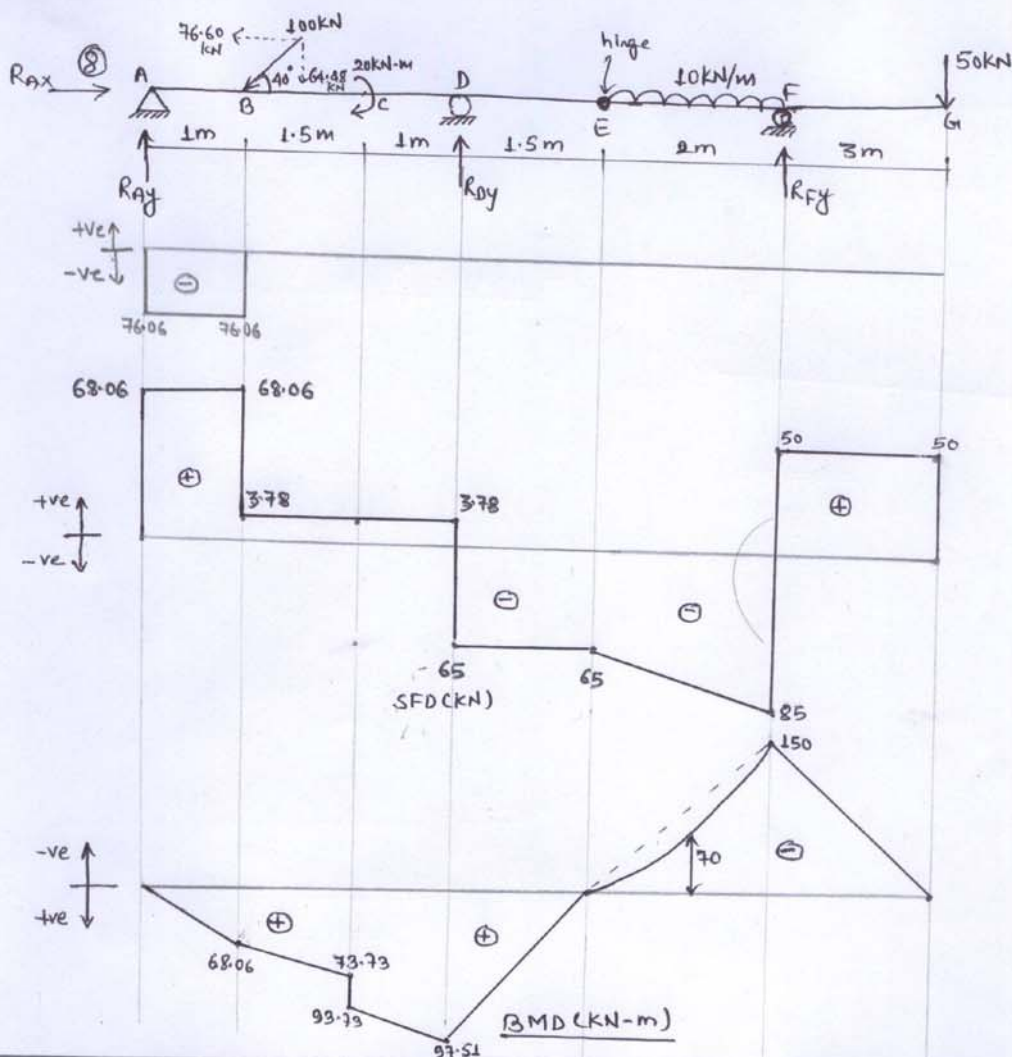
b) In portion CD

$\frac{x}{2.6} = \frac{2-x}{8.66}$



$\Rightarrow x = 0.46 \text{ m}$

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Calculation of  $R_x$

$(\rightarrow) \sum F_x = 0$   
 $R_{Ax} - 100 \cos 40 = 0$   
 $R_{Ax} = 76.60 \text{ kN} (\rightarrow)$   
 $(\uparrow) \sum M_{E, \text{right}} = 0$   
 $-10 \times 2 \times 1 + 2R_{Fy} - 50 \times 5 = 0$   
 $R_{Fy} = 135 \text{ kN} (\uparrow)$   
 $(+\circlearrowleft) \sum M_A = 0$   
 $-64.28 \times 1 - 20 + 3.5R_{Dy} - 10 \times 2 \times 6 + 135 \times 7 - 50 \times 10 = 0$   
 $R_{Dy} = -68.78 \text{ kN}$   
 $= 68.78 \text{ kN} (\downarrow)$

AF Calculation

$A F_{AB} = -76.60 \text{ N}$   
 $(\uparrow) \sum F_y = 0$   
 $R_{Ay} - 64.28 - 68.78 - 10 \times 2 + 135 - 50 = 0$   
 $\Rightarrow R_{Ay} = 68.06 \text{ kN} (\uparrow)$

Shear force caln ( $\uparrow$  +ve)

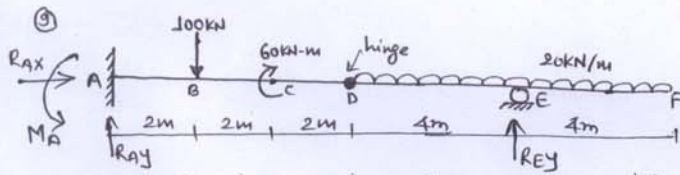
$V_{AL} = 0, V_{AR} = R_{Ay} = 68.06 \text{ kN}$   
 $V_{BL} = V_{AR} = 68.06 \text{ kN}$   
 $V_{BR} = V_{AR} - 64.28 = 3.78 \text{ kN}$   
 $V_{DL} = 3.78 \text{ kN}$   
 $V_{DR} = 3.78 - 68.78 = -65 \text{ kN}$   
 $V_E = -65 \text{ kN}$   
 $V_{FL} = V_E - 10 \times 2 = -85 \text{ kN}$   
 $V_{FR} = V_{FL} + 135 = 50 \text{ kN}$   
 $V_{GL} = V_{FR} = 50 \text{ kN}$   
 $V_{GR} = V_{GL} - 50 = 0$

BM Calculation ( $\curvearrowright$  +ve)

$M_A = 0$   
 $M_B = 68.06 \times 1 = 68.06 \text{ kN-m}$   
 $M_{CL} = 68.06 \times 2.5 - 64.28 \times 1.5 = 73.73 \text{ kN-m}$   
 $M_{CR} = 73.73 + 20 = 93.73 \text{ kN-m}$   
 $M_D = 68.06 \times 3.5 - 64.28 \times 2.5 + 20 = 97.51 \text{ kN-m}$   
 $M_E = 68.06 \times 5 - 64.28 \times 4 + 20 - 68.78 \times 1.5 = 0$   
 $M_F = -50 \times 3 = -150 \text{ kN-m}$   
 $M_G = 0$   
 $M_{EF, \text{mid}} = -50 \times (3+1) + 135 \times 1 - 10 \times 1 \times \frac{1}{2}$   
 $= -70 \text{ kN-m}$

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Calculation of support reaction

$$\sum F_x = 0$$

$$R_{Ax} = 0$$

$$\sum M_{D, \text{right}} = 0$$

$$4R_{Ey} - 20 \times 8 \times \frac{8}{2} = 0$$

$$R_{Ey} = 160 \text{ kN}$$

$$\sum F_y = 0$$

$$R_{Ay} + R_{Ey} - 100 - 20 \times 8 = 0$$

$$R_{Ay} = 100 \text{ kN}$$

$$\sum M_A = 0$$

$$M_A - 100 \times 2 + 60 - 20 \times 8 \times (6+4) + 160R_{Ey} = 0$$

$$M_A = 260 \text{ kN-m}$$

Shear force calculation ( $\uparrow$  +ve  $\downarrow$ )

$$V_{AL} = 0, V_{AR} = 100 \text{ kN}$$

$$V_{BL} = V_{AR} = 100 \text{ kN}$$

$$V_{BR} = V_{BL} - 100 = 0$$

$$V_D = V_{BR} = 0$$

$$V_{EL} = V_D - 20 \times 4 = -80 \text{ kN}$$

$$V_{ER} = V_{EL} + 160 = 80 \text{ kN}$$

$$V_F = V_{ER} - 20 \times 4 = 0$$

Bending Moment calculation ( $\curvearrowright$  +ve  $\curvearrowleft$ )

$$M_{AL} = 0$$

$$M_{AR} = -260 \text{ kN-m}$$

$$M_B = -260 + 100 \times 2 = -60 \text{ kN-m}$$

$$M_{CL} = -260 + 100 \times 4 - 100 \times 2 = -60 \text{ kN-m}$$

$$M_{CR} = M_{CL} + 60 = 0$$

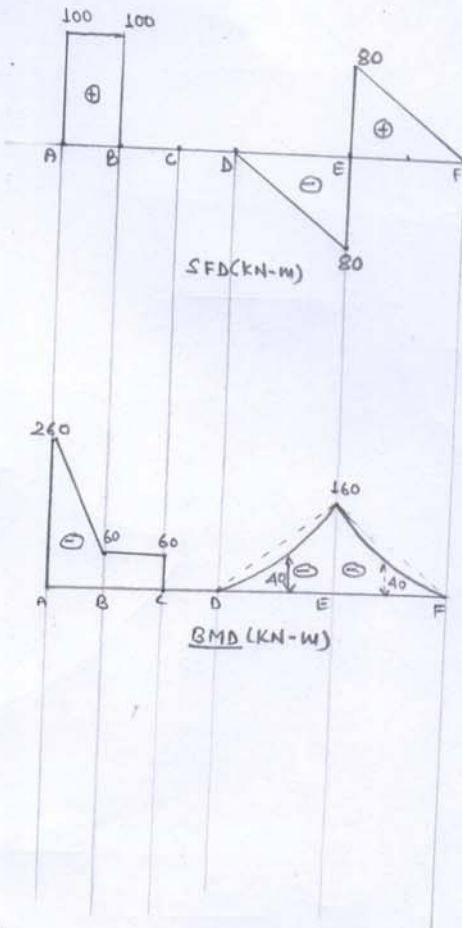
$$M_D = -260 + 100 \times 6 - 100 \times 4 + 60 = 0$$

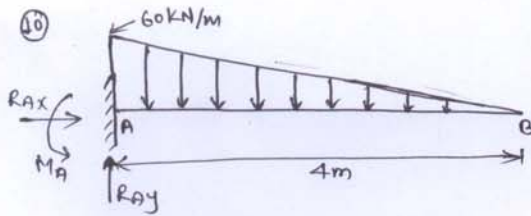
$$M_E = -20 \times 4 \times 2 = -160 \text{ kN-m (from right side cal)}$$

$$M_F = 0$$

$$M_{DE, \text{mid}} = -260 + 100 \times 8 - 100 \times 6 + 60 - 20 \times 2 \times 1 = -40 \text{ kN-m}$$

$$M_{EF, \text{mid}} = -20 \times 2 \times 1 = -40 \text{ kN-m}$$





Calculation of Support reaction

$(\rightarrow) \sum F_x = 0$

$R_{Ax} = 0$

$(\uparrow) \sum F_y = 0$

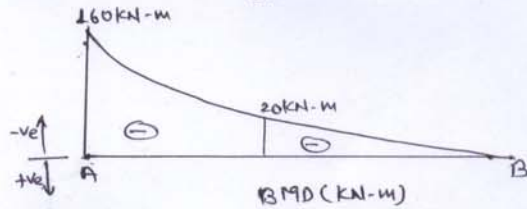
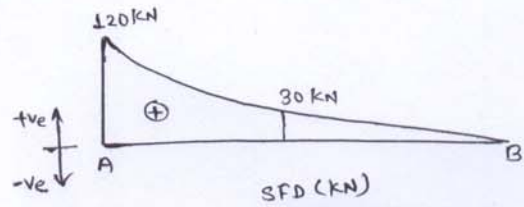
$R_{Ay} - \frac{1}{2} \times 60 \times 4 = 0$

$R_{Ay} = 120 \text{ kN}$

$(\curvearrowright) \sum M_A = 0$

$M_A - \frac{1}{2} \times 60 \times 4 \times \frac{1}{3} \times 4 = 0$

$M_A = 160 \text{ kN-m}$



Shear Force Calculation ( $\uparrow$  tve,  $\downarrow$  -ve)

$V_{AL} = 0, V_{AR} = 120 \text{ kN}$

$V_B = 0$

$V_{AB, \text{mid}} = \frac{1}{2} \times 30 \times 2 = 30 \text{ kN}$

Bending moment calculation ( $\curvearrowright$  tve,  $\curvearrowleft$  -ve)

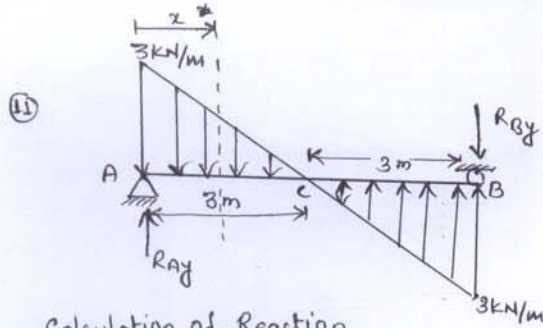
$M_{AL} = 0$

$M_{AR} = -160 \text{ kN-m}$

$M_B = -160 + 120 \times 4 - \frac{1}{2} \times 60 \times 4 \times \frac{2}{3} \times 4$   
 $= 0$

$M_{AB, \text{mid}} = -\frac{1}{2} \times 30 \times 2 \times \frac{1}{3} \times 2$   
 $= -20 \text{ kN-m}$

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Calculation of Reaction

$\sum M_A = 0$

$$-\frac{1}{2} \times 3 \times 3 \times \frac{1}{3} \times 3 + \frac{1}{2} \times 3 \times 3 \times (3 + \frac{2}{3} \times 3) - 6R_{By} = 0$$

$R_{By} = 3 \text{ kN}$

$\sum F_y = 0$

$$R_{Ay} - \frac{1}{2} \times 3 \times 3 - R_{By} + \frac{1}{2} \times 3 \times 3 = 0$$

$\therefore R_{Ay} = 3 \text{ kN}$

Shear force calculation ( $\uparrow$  +ve,  $\downarrow$  -ve)

$V_{AL} = 0$

$V_{AR} = 3 \text{ kN}$

$V_C = 3 - \frac{1}{2} \times 3 \times 3 = -1.5 \text{ kN}$

$V_{BL} = V_C + \frac{1}{2} \times 3 \times 3 = 3 \text{ kN}$

$V_{BR} = V_{BL} - 3 = 0$

$V_{AC, \text{mid}} = V_{AR} - 1.5 \times 1.5 - \frac{1}{2} \times 1.5 \times 1.5 = -0.375$

$V_{CB, \text{mid}} = V_C + \frac{1}{2} \times 1.5 \times 1.5 = -0.375 \text{ kN}$

Bending Moment Calculation ( $\curvearrowright$  +ve,  $\curvearrowleft$  -ve)

$M_A = 0$

$M_{AC, \text{mid}} = 3 \times 1.5 - 1.5 \times 1.5 \times \frac{1.5}{2} - \frac{1}{2} \times 1.5 \times 1.5 \times \frac{2}{3} \times 1.5 = 1.687 \text{ kN-m}$

$M_C = 3 \times 3 - \frac{1}{2} \times 3 \times 3 \times \frac{2}{3} \times 3 = 0$

$M_{CB, \text{mid}} = 3 \times 4.5 - \frac{1}{2} \times 3 \times 3 \times (1.5 + \frac{2}{3} \times 3) + \frac{1}{2} \times 1.5 \times 1.5 \times \frac{1}{3} \times 1.5 = -1.687 \text{ kN-m}$

$M_B = 3 \times 6 - \frac{1}{2} \times 3 \times 3 \times (3 + \frac{2}{3} \times 3) + \frac{1}{2} \times 3 \times 3 \times \frac{1}{3} \times 3 = 0$

For SF zero point in portion AC:

$V_x = 3 - \frac{1}{2} \times x \times x = 0$

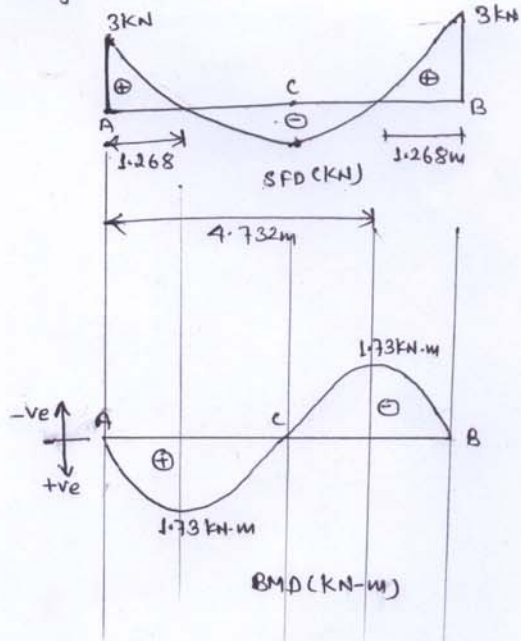
$V_x = 3 - \frac{1}{2} \times x \times x - (3-x) \times x = 0$

$\Rightarrow x = 4.732 \text{ m}, 1.268 \text{ m}$

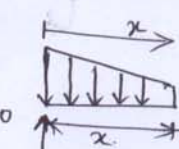
$x^2 - 3x + 6 = 0$

$x = \frac{3 \pm \sqrt{9 - 24}}{2}$

$x = \frac{3 \pm \sqrt{-15}}{2}$



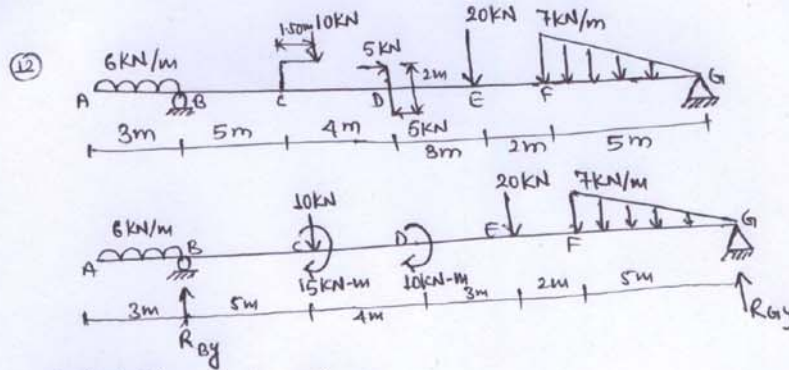
Similarly max. BM in portion BC is  $M_{\text{max}} = -1.73 \text{ kN-m}$



$M_{\text{max}} = 3 \times 1.268 - 3 \times 1.268^2 / 2 + 1.268^3 / 6 = 1.73 \text{ kN-m}$



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Calculation of Reaction:-  
(+)  $\sum M_B = 0$

$$6 \times 3 \times \frac{3}{2} - 10 \times 5 - 15 - 10 - 20 \times 12 - \frac{1}{2} \times 5 \times 7 \times (14 + \frac{1}{3} \times 5) + 19 R_{gy} = 0$$

$$R_{gy} = 29.58 \text{ kN}$$

(+)  $\sum F_y = 0$

$$R_{By} + R_{gy} - 6 \times 3 - 10 - 20 - \frac{1}{2} \times 7 \times 5 = 0$$

$$R_{By} = 35.92 \text{ kN}$$

Shear Force Calculation ( $\uparrow$  +ve  $\downarrow$ )

$$V_A = 0$$

$$V_{BL} = V_A - 6 \times 3 = -18 \text{ kN}$$

$$V_{BR} = V_{BL} + 35.92 = 17.92 \text{ kN}$$

$$V_{CL} = V_{BR} = 17.92 \text{ kN}$$

$$V_{CR} = V_{CL} - 10 = 7.92 \text{ kN}$$

$$V_{DL} = V_{CR} = 7.92 \text{ kN}$$

$$V_{DR} = V_{DL} - 20 = -12.08 \text{ kN}$$

$$V_F = V_{DR} = -12.08 \text{ kN}$$

$$V_{GL} = V_F - \frac{1}{2} \times 7 \times 5 = -29.58 \text{ kN}$$

$$V_{GR} = V_{GL} + R_{gy} = 0$$

Bending Moment Calculation ( $\curvearrowright$  +ve  $\curvearrowleft$ )

$$M_A = 0$$

$$M_B = -6 \times 3 \times \frac{3}{2} = -27 \text{ kN-m}$$

$$M_{CL} = -6 \times 3 \times (5 + \frac{3}{2}) + 35.92 \times 5 = 62.6 \text{ kN-m}$$

$$M_{CR} = M_{CL} + 15 = 77.60 \text{ kN-m}$$

$$M_{DL} = -6 \times 3 \times (9 + \frac{3}{2}) + 35.92 \times 9 - 10 \times 4 + 15 = 109.28 \text{ kN-m}, M_{DR} = M_{DL} + 10 = 119.28 \text{ kN-m}$$

$$M_E = -6 \times 3 \times (12 + \frac{3}{2}) + 35.92 \times 12 - 10 \times 7 + 15 + 10 = 143.04 \text{ kN-m}$$

$$M_F = -6 \times 3 \times (14 + \frac{3}{2}) + 35.92 \times 14 - 10 \times 9 + 15 + 10 - 20 \times 2 = 118.88 \text{ kN-m} \text{ (It's better to calculate from right)}$$

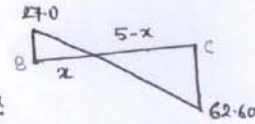
$$M_G = -6 \times 3 \times (19 + \frac{3}{2}) + 35.92 \times 19 - 10 \times 14 + 15 + 10 - 20 \times 7 - \frac{1}{2} \times 7 \times 5 \times \frac{2}{3} \times 5 = 0$$

$$V_{FG, \text{mid}} = -29.58 + \frac{1}{2} \times 3.5 \times 2.5 = -25.20 \text{ kN (from right side cal)}$$

$$M_{AB, \text{mid}} = -6 \times 1.5 \times \frac{1.5}{2} = -6.75 \text{ kN-m}$$

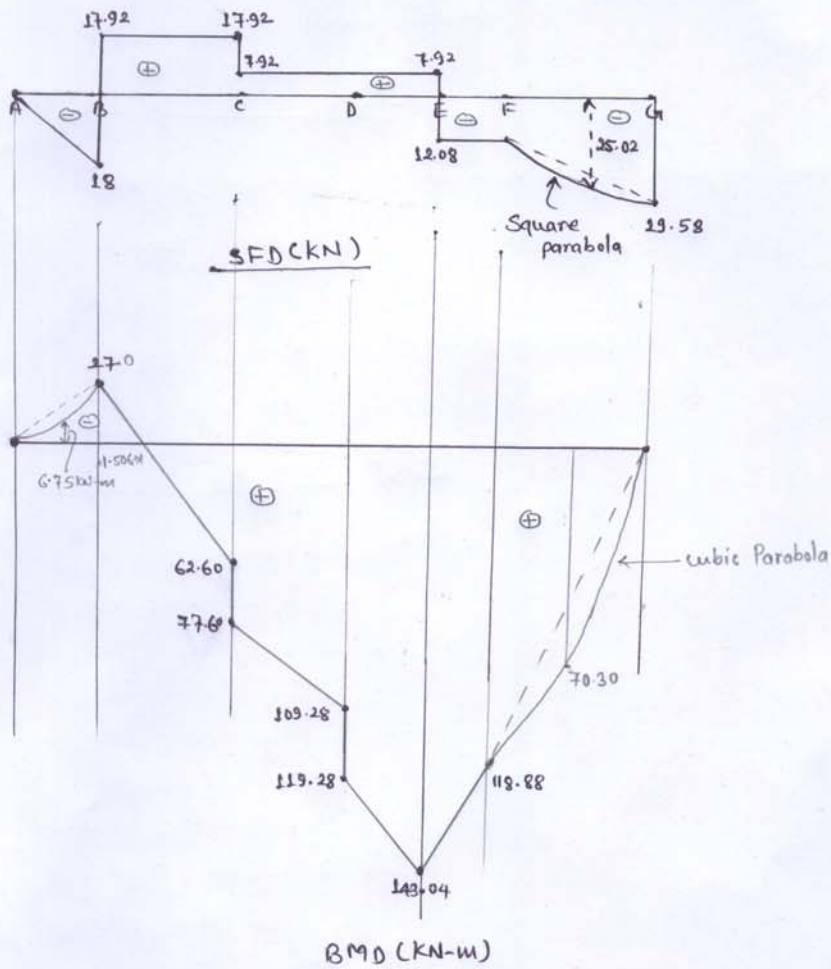
$$M_{FG, \text{mid}} = 29.58 \times 2.5 - \frac{1}{2} \times 3.5 \times 2.5 \times \frac{1}{3} \times 2.5 = 70.3 \text{ kN-m}$$

For Point of contraflexure in BC

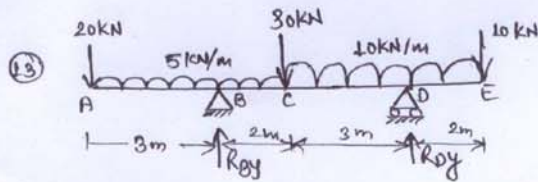


$$\frac{x}{27} = \frac{5-x}{62.6}$$

$$\Rightarrow x = 1.506 \text{ m}$$



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Calculation of reactions:-

$\sum M_B = 0$   
 $20 \times 3 + 5 \times 5 \times 0.5 - 30 \times 2 - 10 \times 5 \times (2 + 2.5) - 10 \times 7 + R_{Dy} \times 5 = 0$   
 $R_{Dy} = 56.50 \text{ kN}$

$\sum F_y = 0$   
 $R_{By} + R_{Dy} - 20 - 30 - 10 - 5 \times 5 - 10 \times 5 = 0$   
 $R_{By} = 78.50 \text{ kN}$

SF calculation ( $\uparrow$  +ve,  $\downarrow$  -ve)

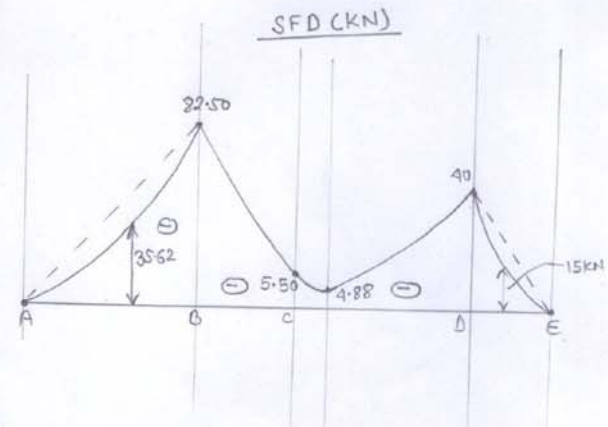
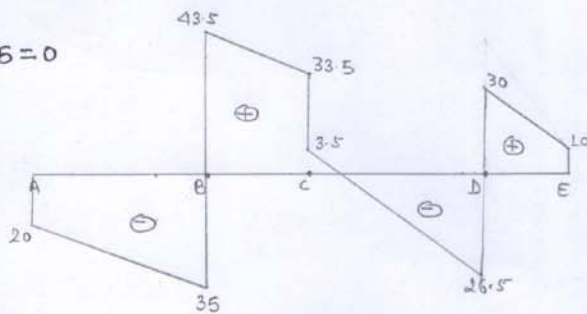
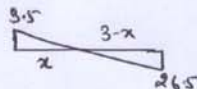
$V_A = 0$   
 $V_{AR} = -20 \text{ kN}$   
 $V_{BL} = V_{AR} - 5 \times 3 = -35 \text{ kN}$   
 $V_{BR} = V_{BL} + 78.5 = 43.5 \text{ kN}$   
 $V_{CL} = V_{BR} - 5 \times 2 = 33.5 \text{ kN}$   
 $V_{CR} = V_{CL} - 30 = 3.5 \text{ kN}$   
 $V_{DL} = V_{CR} - 10 \times 3 = -26.5 \text{ kN}$   
 $V_{DR} = V_{DL} + 56.5 = 30 \text{ kN}$   
 $V_{EL} = V_{DR} - 10 \times 2 = 10 \text{ kN}$   
 $V_{ER} = V_{EL} - 10 = 0$

BM calculation ( $\curvearrowright$  +ve,  $\curvearrowleft$  -ve)

$M_A = 0$   
 $M_B = -20 \times 3 - 5 \times 3 \times \frac{3}{2} = -82.50 \text{ kN-m}$   
 $M_C = -20 \times 5 - 5 \times 5 \times \frac{5}{2} + 78.5 \times 2 = -5.50 \text{ kN-m}$   
 $M_D = -10 \times 2 - 10 \times 2 \times 1 = -40 \text{ kN-m (from right)}$   
 $M_E = 0$ ,  $M_{AB, \text{mid}} = -20 \times 1.5 - 5 \times 1.5 \times \frac{1.5}{2} = -35.62 \text{ kN-m}$   
 $M_{DE, \text{mid}} = -10 \times 1 - 10 \times 1 \times \frac{1}{2} = -15 \text{ kN-m}$

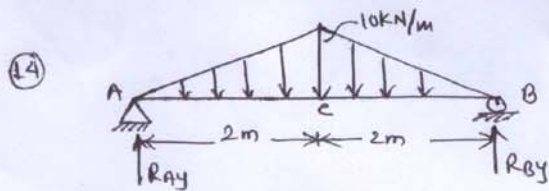
SF zero point in portion CD:

$\frac{x}{3.5} = \frac{3-x}{26.5}$   
 $x = 0.35 \text{ m}$



$\therefore \text{max. BM } (M_{\text{max}}) = -20 \times 5.35 + 78.5 \times 2.35 - 30 \times 0.35 - 5 \times 5 \times (2.5 + 0.35) - 10 \times 0.35 \times \frac{0.35}{2}$   
 $= -4.88 \text{ kN-m}$





Calculation of reaction

$$R_{Ay} = R_{By} = \frac{1}{2} \left[ \frac{1}{2} \times 2 \times 10 + \frac{1}{2} \times 2 \times 10 \right]$$

$$= 10 \text{ kN}$$

Calculation of SF ( $\uparrow$  +ve  $\downarrow$ )

$$V_{AL} = 0, V_{AR} = 10 \text{ kN}$$

$$V_C = V_{AR} - \frac{1}{2} \times 2 \times 10 = 0$$

$$V_{BL} = V_C - \frac{1}{2} \times 2 \times 10 = -10 \text{ kN}$$

$$V_{BR} = V_{BL} + 10 = 0$$

$$V_{AC, \text{mid}} = 10 - \frac{1}{2} \times 1 \times 5 = 7.5 \text{ kN}$$

$$V_{BC, \text{mid}} = -10 + \frac{1}{2} \times 1 \times 5 = -7.5 \text{ kN}$$

Calculation of Bending Moment ( $\curvearrowright$  +ve)

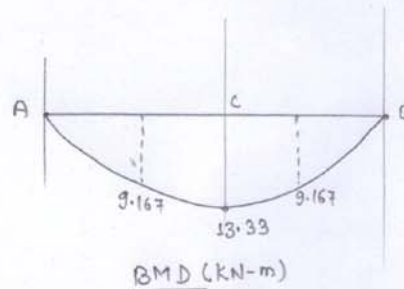
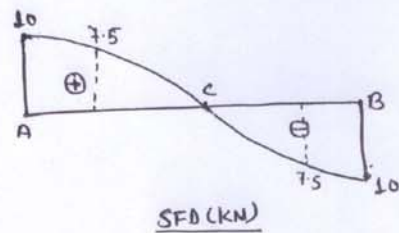
$$M_A = 0$$

$$M_C = 10 \times 2 - \frac{1}{2} \times 2 \times 10 \times \frac{1}{3} \times 2 = 13.33 \text{ kN-m}$$

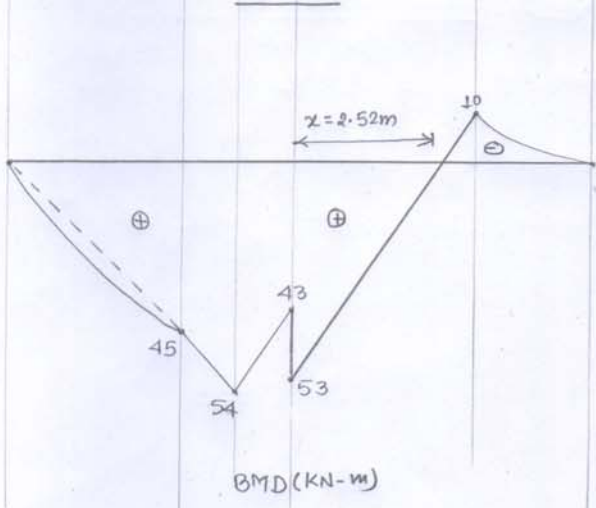
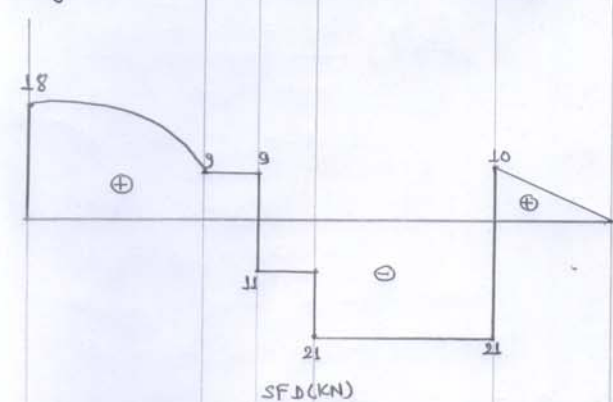
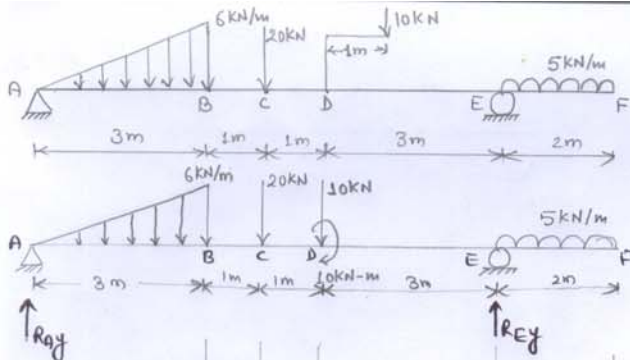
$$M_B = 0$$

$$M_{AC, \text{mid}} = 10 \times 1 - \frac{1}{2} \times 1 \times 5 \times \frac{1}{3} \times 1 = 9.167 \text{ kN-m}$$

$$M_{BC, \text{mid}} = 10 \times 1 - \frac{1}{2} \times 1 \times 5 \times \frac{1}{3} \times 1 = 9.167 \text{ kN-m}$$



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Calculation of Reaction

$\sum M_A = 0$   
 $8R_{EY} - \frac{1}{2} \times 3 \times 6 \times \frac{2}{3} \times 9 - 20 \times 4 - 10 - 10 \times 5 - 5 \times 2 \times 9 = 0$   
 $R_{EY} = 31 \text{ kN}$   
 $(\uparrow) \sum F_y = 0$   
 $R_{AY} + R_{EY} - \frac{1}{2} \times 3 \times 6 - 20 - 10 - 5 \times 2 = 0$   
 $R_{AY} = 18 \text{ kN}$

Shear Force Calculation ( $\uparrow$  +ve  $\downarrow$ )

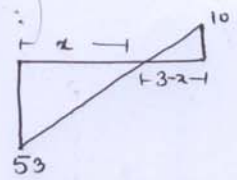
$V_{AL} = 0, V_{AR} = 18 \text{ kN}$   
 $V_B = V_{AR} - \frac{1}{2} \times 3 \times 6 = 9 \text{ kN}$   
 $V_{AB, \text{mid}} = 18 - \frac{1}{2} \times 1.5 \times 3 = 15.75 \text{ kN}$   
 $V_{CL} = V_B = 9 \text{ kN}$   
 $V_{CR} = V_{CL} - 20 = -11 \text{ kN}$   
 $V_{DL} = V_{CR} = -11 \text{ kN}$   
 $V_{DR} = V_{DL} - 10 = -21 \text{ kN}$   
 $V_{EL} = V_{DR} = -21 \text{ kN}$   
 $V_{ER} = V_{EL} + 31 = 10 \text{ kN}$   
 $V_F = V_{ER} - 5 \times 2 = 0$

Bending Moment Calculation ( $\curvearrowright$  +ve)

$M_A = 0$   
 $M_B = 18 \times 3 - 9 \times \frac{1}{3} \times 3 = 45 \text{ kN-m}$   
 $M_{AB, \text{mid}} = 18 \times 1.5 - \frac{1}{2} \times 1.5 \times 3 \times \frac{1}{3} \times 1.5 = 25.875 \text{ kN-m}$   
 $M_C = 18 \times 4 - \frac{1}{2} \times 3 \times 6 \times (1 + \frac{1}{3} \times 3) = 54 \text{ kN-m}$   
 $M_{DL} = 18 \times 5 - \frac{1}{2} \times 3 \times 6 \times (2 + \frac{1}{3} \times 3) - 20 \times 1 = 43 \text{ kN-m}$   
 $M_{DR} = 43 + 10 = 53 \text{ kN-m}$   
 $M_E = -5 \times 2 \times 1 = -10 \text{ kN-m}$   
 $M_F = 0$   
 $M_{EF, \text{mid}} = -5 \times 1 \times \frac{1}{2} = -2.5 \text{ kN-m}$

For point of contraflexure in DE

$\frac{x}{53} = \frac{0-10}{10}$   
 $\Rightarrow x = 2.52 \text{ m}$



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